

On the constant surface density in dark matter galaxies and interstellar molecular clouds

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Cumulative observational evidence including recent findings confirm that the surface density of dark matter (DM) halos $\mu_{0D} = r_0 \rho_0$ where r_0 and ρ_0 are the halo core radius and central density, respectively, is nearly **constant** and independent of galaxy luminosity for a high number of galactic systems (spirals, dwarf irregular and spheroidals, elliptics) spanning over 14 magnitudes in luminosity and of different Hubble types. We point out that this **universal** relation is of the same kind as the Larson relation for the average surface or column density μ_{0D} of interstellar molecular clouds of size r_0 (valid irrespective of the cloud type and composition over more than four orders of magnitude of scales $0.001 \text{ pc} < r_0 < 100 \text{ pc}$). Remarkably, its numerical value $\mu_{0D} \simeq 140 M_\odot/\text{pc}^2 = (18.6 \text{ MeV})^3$ is approximately **the same** (up to a factor of two) in all these systems. A simple physical explanation of the constance of μ_{0D} based on the virial theorem and the extensivity property of the total energy is given. The action of self-gravity (both of baryonic and dark matter) must be the origin of the **universal** character of μ_{0D} for all these cored bounded virialized systems, μ_{0D} being a dynamical scale determined by gravitational clustering. We theoretically compute μ_{0D} and the density profile for small scales from the linearized Boltzmann-Vlasov equation for self-gravitating DM. We solve this equation during the matter dominated era in the Gilbert form. We use the appropriate initial conditions including the primordial inflationary power spectrum and the evolution through the radiation dominated era. The theoretically derived density profiles $\rho_{lin}(r)$ turn to be **cored** and the resulting value for μ_{0D} is remarkably close to the observations. Our density profile $\rho_{lin}(r)$ decreases as $r^{-1-n_s/2}$ for small scales where $n_s \simeq 0.964$ is the **primordial** spectral index. This scaling is in remarkably agreement with the empirical behaviour recently found observationally: $r^{-1.6 \pm 0.4}$. The value of μ_{0D} implies that the DM particle mass is in the keV scale. We consider both fermions and bosons as DM particles decoupling either ultrarelativistically or non-relativistically. Our results do **not** use any particle physics model and vary slightly with the statistics of the DM particle. This analysis provides further evidence for the mass scale of the (cold) dark matter particle in the keV range.

I. OBSERVATIONAL EVIDENCES, THE VIRIAL THEOREM AND THE EXTENSIVITY OF ENERGY

Among the scaling laws put forward by Larson [1] for interstellar molecular clouds in our Galaxy there is the relation between the cloud size r_0 and the average density ρ_0 :

$$r_0 \rho_0 = \mu_{0D}$$

where the column density μ_{0D} is approximately a **constant** over more than four orders of magnitude of scales $0.001 \text{ pc} < r_0 < 100 \text{ pc}$. The values given in ref.[1] are:

$$\mu_{0D} = 10.5 \cdot 10^{21} \frac{m_{H_2}}{\text{cm}^2} = 162 \frac{M_\odot}{\text{pc}^2}$$

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where m_{H_2} stands for the mass of the Hydrogen molecule, main constituent of the interstellar clouds. Recent data averaged over high density regions of Taurus give [2]

$$\mu_{0D} = 5.14 \cdot 10^{21} \frac{m_{H_2}}{\text{cm}^2} = 80 \frac{M_\odot}{\text{pc}^2}$$

The mean density of structures in the ISM vary between 10 and 10^5 atoms/cm³, significantly above the mean ISM density which is about 0.1 atoms/cm³ or $1.6 \cdot 10^{-25}$ g/cm³.

Growing recent findings point towards the **same relation** for the dark matter (DM) surface density μ_{0D} in galaxies [3, 4, 7]. Namely, that the product $\mu_{0D} \equiv r_0 \rho_0$ where r_0 and ρ_0 are the halo core radius and central density, respectively, is nearly **constant**, over a large number of galaxies of different kinds. r_0 varies by two orders of magnitude [3, 4, 7]:

$$0.3 \text{ kpc} < r_0 < 30 \text{ kpc} \quad \text{and} \quad 10^{-25} \text{ g/cm}^3 \leq \rho_0 \leq 6 \times 10^{-23} \text{ g/cm}^3. \quad (1.1)$$

This finding relates to data sets (high quality rotation curves, kinematics, galaxy-galaxy weak lensing signals) for many galactic systems spanning over 14 magnitudes in luminosity and of different Hubble type, dwarf disk and spheroidals, spirals, ellipticals. In spite of their different properties, μ_{0D} in galaxies is essentially **independent** of their luminosity and mass.

Remarkably enough, μ_{0D} takes in **all** these systems (molecular clouds and galaxy systems) approximatively the same value [1, 4, 5, 6]

$$\mu_{0D} \simeq 140 \frac{M_\odot}{\text{pc}^2} = 6400 \text{ MeV}^3 = (18.6 \text{ MeV})^3. \quad (1.2)$$

We give in this section a simple physical explanation of the constance of μ_{0D} and its value based on the virial theorem and the extensivity property of the total energy. We derive in the next section the value of μ_{0D} from the resolution of the linearized Boltzmann-Vlasov equation for self-gravitating DM.

Assuming that the matter distribution in the galaxy is characterized by a scale r_0 , the matter density can be written as

$$\rho(r) = \rho_0 F\left(\frac{r}{r_0}\right) \quad , \quad F(0) = 1. \quad (1.3)$$

Algebraic fits to the cored density profile [7, 8] and thermal profiles [10] are particular examples of eq.(1.3). We have for the Burkert [8], Spano [7] and Sérsic profiles [9] (denoted F_B , F_{Sp} and F_S , respectively):

$$F_B(x) = \frac{1}{(1+x)(1+x^2)} \quad , \quad F_{Sp}(x) = \frac{1}{(1+x^2)^{\frac{3}{2}}} \quad , \quad F_S(x) = e^{-x^{\frac{1}{n}}} \quad , \quad x \equiv \frac{r}{r_0} ,$$

where $\frac{1}{2} < n < 10$ fits most of the galaxies in the Sérsic profile. The de Vaucouleurs profile is a particular case of the Sérsic profile for $n = 4$. Notice that both the Burkert and the Spano profiles decay for large distances with the same $1/r^3$ tail as the cuspy NFW profile [12]

$$F_{NFW}(x) = \frac{4}{x(1+x)^2} .$$

The virial theorem for self-gravitating systems states that the total energy E is related to the average potential energy $\langle U \rangle$ and the average kinetic energy $\langle K \rangle$ by [13]

$$E = \frac{1}{2} \langle U \rangle = -\langle K \rangle. \quad (1.4)$$

We can therefore express the total energy E in terms of the average gravitational potential energy as

$$E = -\frac{1}{4} G \int \frac{d^3r d^3r'}{|\vec{r} - \vec{r}'|} \langle \rho(r) \rho(r') \rangle = -\frac{1}{4} G \rho_0^2 r_0^5 \int \frac{d^3x d^3x'}{|\vec{x} - \vec{x}'|} \langle F(x) F(x') \rangle .$$

Hence, the energy divided by the characteristic volume r_0^3 goes as

$$\frac{-E}{r_0^3} \sim G \rho_0^2 r_0^2 = G \mu_{0D}^2. \quad (1.5)$$

Extensivity of the energy requires E/r_0^3 to stay fixed for large values of r_0 [10, 11, 16]. Therefore, μ_{0D} must take the same **constant** value irrespective of the value of r_0 . Eq.(1.5) thus provides a simple explanation of the astronomical observations [1, 3, 4, 7].

We can estimate the average kinetic energy $\langle K \rangle$ as

$$\langle K \rangle = \frac{1}{2} \int d^3r \langle \rho(r) \rangle \langle v^2 \rangle = \frac{1}{2} \rho_0 r_0^3 \langle v^2 \rangle \int d^3x \langle F(x) \rangle \sim \rho_0 r_0^3 \langle v^2 \rangle . \quad (1.6)$$

We thus find in general the average squared velocity $\langle v^2 \rangle$ for any profile eq.(1.3) from eqs.(1.4), (1.5) and (1.6),

$$\langle v^2 \rangle \sim G \mu_{0D} r_0 , \quad (1.7)$$

as in the particular case of the Burkert profile [4]. Eq.(1.7) is verified **both** for molecular clouds and galaxies.

For the self-gravitating gas in thermal equilibrium the above argument eq.(1.5) of the extensivity property of the energy leads to a density $\rho(r)$ that scales as $1/r$ [10].

The quantities r_0 and ρ_0 depend on the particular galaxy (or molecular cloud) chosen and are therefore functions of the past history of the galaxy (or cloud). Instead, the product $\mu_{0D} = r_0 \rho_0$ given by eq.(1.2) is an **universal** number for all galactic systems and molecular clouds and hence independent of the previous history of the system. μ_{0D} can only depend on **universal quantities**. Since μ_{0D} is the same (up to a factor two) for molecular clouds and galaxies, the action self-gravity (both of baryonic and dark matter) must be responsible of its value. As we show in the next section, μ_{0D} is a dynamical scale determined by gravitational clustering. We consider in what follows DM in galaxies.

II. THE SURFACE DENSITY FROM THE LINEARIZED BOLTZMANN-VLASOV EQUATION

The mass density $\rho_{lin}(r)$ can be evaluated theoretically solving the linearized Boltzmann-Vlasov equation for self-gravitating DM in the matter dominated (MD) era. It is convenient to recast such equation as a integral equation, namely the Gilbert equation which is a Volterra equation of second kind [17, 18, 19]. To linear order in perturbations the distribution function of the decoupled particles can be written as

$$f(\vec{x}, \vec{p}; t) = g f_0(p) + F_1(\vec{x}, \vec{p}; t)$$

where \vec{x}, \vec{p} are comoving coordinates, g is the number of internal degrees of freedom of the DM particle, typically $1 \leq g \leq 4$, $f_0(p)$ is the thermal equilibrium unperturbed distribution function at the decoupling temperature T_d normalized by

$$m g \int \frac{d^3p}{(2\pi)^3} f_0(p) = \rho_{DM} = \Omega_M \rho_c , \quad (2.1)$$

$\Omega_M = 0.233$ is the DM fraction, ρ_c is the critical density of the Universe

$$\rho_c = 3 M_{Pl}^2 H_0^2 = (2.518 \text{ meV})^4 , \quad 1 \text{ meV} = 10^{-3} \text{ eV} \quad (2.2)$$

and $H_0 = 1.5028 \cdot 10^{-42} \text{ GeV}$. Terms of order higher than one in F_1 are neglected in the Boltzmann-Vlasov equation. The physical initial conditions at t_{eq} , the beginning of the MD era, are

$$f(\vec{x}, \vec{p}; t_{eq}) = g f_0(p) [1 + \delta(\vec{x}, t_{eq})] , \quad F_1(\vec{x}, \vec{p}; t_{eq}) = g f_0(p) \delta(\vec{x}, t_{eq}) , \quad (2.3)$$

where $\delta(\vec{x}, t_{eq})$ are the density fluctuations by the beginning of the MD era.

It is useful to integrate the fluctuations $F_1(\vec{p}; \vec{x}; t)$ over the momentum \vec{p} ,

$$\Delta(k, t) \equiv m \int \frac{d^3p}{(2\pi)^3} F_1(\vec{k}, \vec{p}; t) \quad \text{where} \quad F_1(\vec{k}, \vec{p}; t) = \int d^3x e^{-i \vec{x} \cdot \vec{k}} F_1(\vec{x}, \vec{p}; t) . \quad (2.4)$$

Its Fourier transform provides the matter density fluctuations in linear approximation today

$$\rho_{lin}(r) = \frac{1}{2 \pi^2 r} \int_0^\infty k dk \sin(k r) \Delta(k, t_{\text{today}}) , \quad (2.5)$$

where as customary we considered a spherical symmetric distribution.

We therefore have as initial conditions,

$$\Delta(k, t_{eq}) = \Omega_M \rho_c V \delta(k, t_{eq}) , \quad (2.6)$$

where $V \sim 1/k_{eq}^3$ is the comoving horizon volume by equilibration. Namely, all fluctuations with $k > k_{eq}$ that were inside the horizon by equilibration are relevant here. More explicitly, $k_{eq} \simeq 42.04 H_0 = 9.88 \text{ Gpc}^{-1}$ [25] and

$$V \simeq \frac{1}{k_{eq}^3} \simeq \frac{f}{H_0^3} \quad \text{where} \quad f \simeq 1.346 \cdot 10^{-5} . \quad (2.7)$$

The present linear treatment is valid for small scales $k > k_{eq}$. Non-linear effects become important for large scales $k < k_{eq}$ and call for the use of the full (non-linear) Boltzmann-Vlasov equation or N -body simulations.

The density fluctuations $\delta(k, t)$ by the end of the RD era can be obtained analytically for subhorizon wavenumbers [25, 26]. The initial conditions for $\delta(k, t)$ in the RD era are the primordial inflationary fluctuations. With such initial conditions and solving the fluid equations for DM during the RD era yields [25, 26]

$$\delta(k, t_{eq}) = \frac{1}{2} A |\phi_k| \left\{ 5 \log \left[\frac{4 \sqrt{2} B k}{k_{eq} (1 + \sqrt{2})^2} \right] + 6 \sqrt{2} - 15 \right\} = \frac{5}{2} A |\phi_k| \log \left(0.2637 B \frac{k}{k_{eq}} \right) , \quad (2.8)$$

where $A \simeq 9.6$ and $B \simeq 0.44$ are constants that follow evolving the fluid equations [26], ϕ_k are the primordial inflationary fluctuations of the newtonian potential [20, 25]

$$|\phi_k| = \sqrt{2} \pi |\Delta_0| \left(\frac{k}{k_0} \right)^{n_s/2-2} , \quad (2.9)$$

$|\Delta_0|$ stands for the primordial power, n_s is the spectral index, and k_0 the pivot wavenumber [20, 21],

$$|\Delta_0| \simeq 4.94 \cdot 10^{-5} \quad , \quad n_s \simeq 0.964 \quad , \quad k_0 = 2 \text{ Gpc}^{-1} . \quad (2.10)$$

It is convenient to define

$$\hat{\Delta}(k, t) \equiv \frac{\Delta(k, t)}{\Delta(k, t_{eq})} \quad (2.11)$$

Then, the Gilbert equation takes the form [19]

$$\begin{aligned} \hat{\Delta}(k, u) - \frac{6}{\alpha} \int_0^u \Pi[\alpha(u - u')] \frac{\hat{\Delta}(k, u')}{[1 - u']^2} du' &= I[\alpha u] \quad \text{where} , \\ \Pi[z] &= \frac{1}{I_2} \int_0^\infty dy y f_0(y) \sin(yz) \quad , \quad I[z] = \frac{1}{I_2} \int_0^\infty dy y f_0(y) \frac{\sin(yz)}{z} \quad \text{and} \\ y &\equiv \frac{p}{T_d} , \quad z \equiv \alpha u , \quad \alpha \equiv \frac{2k}{H_0} \sqrt{\frac{1 + z_{eq}}{\Omega_M}} \frac{T_d}{m} , \\ I_2 &= \int_0^\infty dy y^2 f_0(y) \quad , \quad 1 + z_{eq} = \frac{1}{a_{eq}} \simeq 3200 . \end{aligned} \quad (2.12)$$

u is a dimensionless time variable related to the scale factor by

$$u = 1 - \sqrt{\frac{a_{eq}}{a}} \quad , \quad a(u) = \frac{a_{eq}}{(1 - u)^2} \quad , \quad 0 \leq u \leq u_{\text{today}} = 1 - \sqrt{a_{eq}} \simeq 0.982 \quad , \quad a(\text{today}) = 1 .$$

It follows from the resolution of the Gilbert equation eq.(2.12) that for late times the solution grows as [19]

$$\hat{\Delta}(k, t) \xrightarrow{t \rightarrow t_{\text{today}}} \frac{3}{5} T(k) (1 + z_{eq}) \quad (2.13)$$

where $T(k)$ stands for the transfer function. $\hat{\Delta}(k, t)$ grows proportional to the scale factor in the linear approximation for all $k < k_{fs}$. The free-streaming comoving wavenumber k_{fs} increases with time as $1/\sqrt{1 + z}$.

Notice that $T(0) = 1$ and $T(k \rightarrow \infty) = 0$. $T(k)$ decreases with k according to the characteristic scale given by the free streaming wavenumber k_{fs} where $l_{fs} = \sqrt{6}/k_{fs}$ is the free streaming length [19]. The explicit expression of the comoving free streaming length is [19]

$$l_{fs} = \frac{2\sqrt{3}}{H_0} \sigma_{DM} \sqrt{\frac{1+z_{eq}}{\Omega_M}} \quad , \quad \sigma_{DM}^2 \equiv \frac{1}{3} \langle v^2 \rangle . \quad (2.14)$$

σ_{DM} stands for the primordial comoving squared velocity dispersion of the DM particles. That is, the velocity dispersion computed from the equilibrium thermal distribution $f_0(p)$ which can be expressed as

$$\sigma_{DM} = \sqrt{\frac{I_4}{3 I_2}} \frac{T_d}{m} \quad \text{where} \quad I_4 = \int_0^\infty dy y^4 f_0(y) . \quad (2.15)$$

It is convenient to introduce the variable

$$\gamma \equiv k r_{lin} \quad \text{where} \quad r_{lin} \equiv \frac{l_{fs}}{\sqrt{3}} = \frac{\sqrt{2}}{k_{fs}} , \quad (2.16)$$

and consider the transfer function $T(k)$ as a function of γ . $T(\gamma)$ decreases by an amount of order one for γ increasing by unit. Therefore, its Fourier transform $\rho_{lin}(r)$ eq.(2.5), decreases with r having r_{lin} as characteristic scale.

$T(k)$ is obtained by solving the Gilbert equation (2.12) [19]. We plot in fig. 1 $T(k)$ for Fermions (FD) and Bosons (BE) decoupling ultrarelativistically and for particles decoupling non-relativistically (Maxwell-Boltzmann statistics).

The dark matter density eq.(2.1) can also be expressed as an integral over y as

$$\rho_{DM} = \frac{m}{2\pi^2} g T_d^3 I_2 . \quad (2.17)$$

The covariant decoupling temperature T_d can be related to the effective number of UR degrees of freedom at decoupling g_d and the photon temperature today T_{cmb} by using entropy conservation [28]:

$$T_d = \left(\frac{2}{g_d} \right)^{\frac{1}{3}} T_{cmb} , \quad \text{where} \quad T_{cmb} = 0.2348 \text{ meV} . \quad (2.18)$$

We obtain the amplitude $\Delta(k, t)$ today by inserting eqs. (2.6), (2.7), (2.8), (2.9) and (2.13) into eq.(2.11) for $t = t_{\text{today}}$

$$\Delta(k, t_{\text{today}}) = \frac{9\pi}{\sqrt{2}} \frac{M_{Pl}^2}{H_0} \Omega_M f A (1+z_{eq}) |\Delta_0| T(k) \left(\frac{k}{k_0} \right)^{n_s/2-2} \log \left(c \frac{k}{k_{eq}} \right) . \quad (2.19)$$

where $c = 0.11604$. Inserting eq.(2.19) into eq.(2.5) yields the density profile today,

$$\begin{aligned} \rho_{lin}(r) &= \frac{27\sqrt{2}}{5\pi} \frac{\Omega_M^2 M_{Pl}^2 H_0}{\sigma_{DM}^2} f |\Delta_0| (k_0 r_{lin})^{2-n_s/2} \frac{1}{r} \int_0^\infty d\gamma \gamma^{n_s/2-1} \log \left(\frac{c\gamma}{k_{eq} r_{lin}} \right) \sin \left(\gamma \frac{r}{r_{lin}} \right) T(\gamma) , \\ r_{lin} \rho_{lin}(0) &= \frac{27\sqrt{2}}{5\pi} \frac{\Omega_M^2 M_{Pl}^2 H_0}{\sigma_{DM}^2} f |\Delta_0| (k_0 r_{lin})^{2-n_s/2} \int_0^\infty d\gamma \gamma^{n_s/2} \log \left(\frac{c\gamma}{k_{eq} r_{lin}} \right) T(\gamma) , \end{aligned} \quad (2.20)$$

It is very useful to relate the free streaming length to the phase-space density ρ/σ^3 [14, 15, 22]. ρ/σ^3 is invariant under the cosmological expansion and decreases due to gravitational clustering. The phase-space density before structure formation (ρ_{DM}/σ_{DM}^3) and today can be related as [15]

$$\frac{\rho_s}{\sigma_s^3} = \frac{1}{Z} \frac{\rho_{DM}}{\sigma_{DM}^3} . \quad (2.21)$$

where the value today

$$\frac{\rho_s}{\sigma_s^3} \sim 5 \times 10^3 \frac{\text{keV/cm}^3}{(\text{km/s})^3} = (0.18 \text{ keV})^4 , \quad (2.22)$$

follows from dSphs observations [23], and according to N -body simulations $1 < Z < 1000$ [24] .

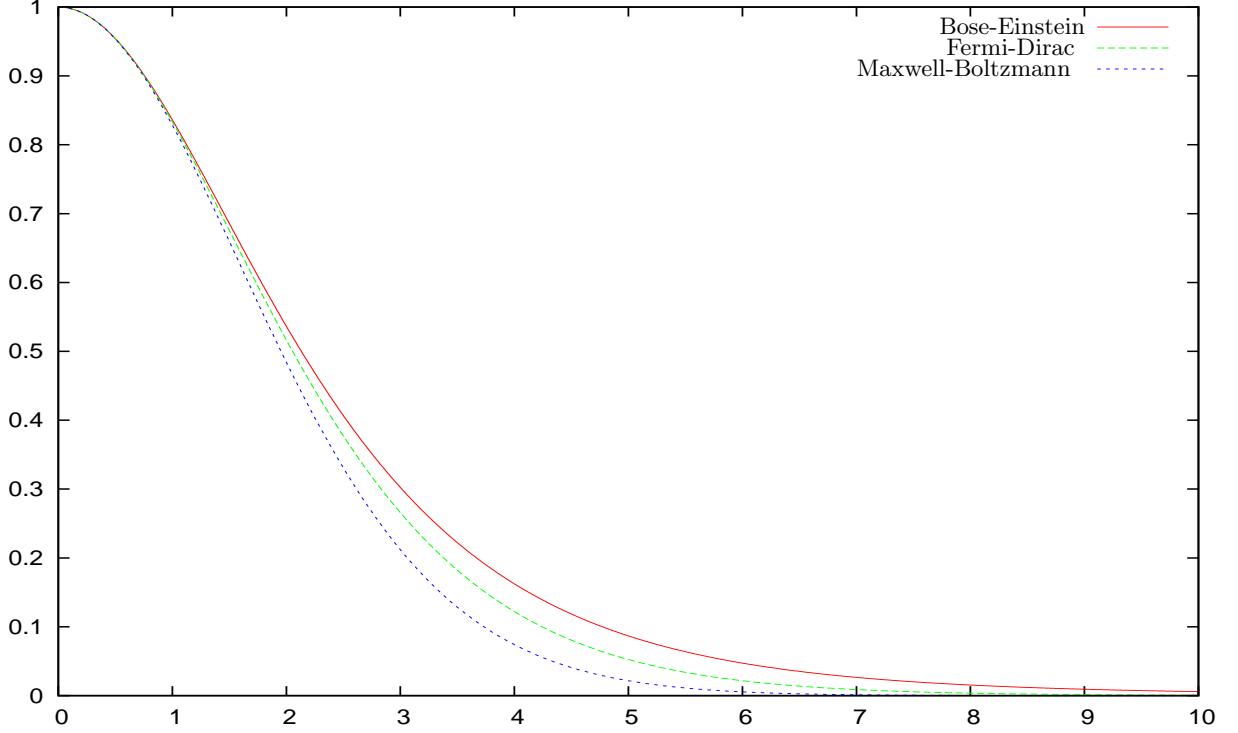


FIG. 1: The transfer function $T(k)$ vs. $\gamma = k r_{lin}$ for Fermions and Bosons decoupling ultrarelativistically and for particles decoupling non-relativistically (Maxwell-Boltzmann statistics). We see that $T(k)$ decays for increasing k with a characteristic scale $\sim 1/r_{lin} \sim k_{fs}$ [see eq.(2.16)].

Combining eqs.(2.15), (2.17), (2.18) and (2.21) we can express m and g_d as

$$m^4 = \frac{2 \pi^2}{3 \sqrt{3}} \frac{Z}{g} \frac{\rho_s}{\sigma_s^3} \frac{I_4^{\frac{3}{2}}}{I_2^{\frac{5}{2}}} \quad , \quad m = 0.2504 \left(\frac{Z}{g} \right)^{\frac{1}{4}} \frac{I_4^{\frac{3}{8}}}{I_2^{\frac{5}{8}}} \text{ keV} \quad , \quad (2.23)$$

$$g_d = \frac{2^{\frac{1}{4}}}{3^{\frac{11}{8}} \pi^{\frac{3}{2}}} \frac{g^{\frac{3}{4}}}{\Omega_{DM}} \frac{T_{cmb}^3}{M_{Pl}^2 H_0^2} \left(\frac{Z \rho_s}{\sigma_s^3} \right)^{\frac{1}{4}} (I_2 I_4)^{\frac{3}{8}} = 35.96 Z^{\frac{1}{4}} g^{\frac{3}{4}} (I_2 I_4)^{\frac{3}{8}} \quad . \quad (2.24)$$

For example, for fermions and bosons that decouple ultrarelativistically at thermal equilibrium eqs.(2.23) and (2.24) yield [15]

$$m = \left(\frac{Z}{g} \right)^{\frac{1}{4}} \text{ keV} \begin{cases} 0.568 & \text{Fermions} \\ 0.484 & \text{Bosons} \end{cases} \quad , \quad g_d = g^{\frac{3}{4}} Z^{\frac{1}{4}} \begin{cases} 155 & \text{Fermions} \\ 180 & \text{Bosons} \end{cases} \quad . \quad (2.25)$$

Notice that $1 < Z^{\frac{1}{4}} < 5.6$ for $1 < Z < 1000$.

We obtain the primordial DM dispersion velocity σ_{DM} from eqs. (2.1), (2.2) and (2.21) [15],

$$\sigma_{DM} = \left(3 M_{Pl}^2 H_0^2 \Omega_{DM} \frac{1}{Z} \frac{\sigma_s^3}{\rho_s} \right)^{\frac{1}{3}} \quad (2.26)$$

This expression is valid for **any kind** of DM particles. We find using eq.(2.14) for the free streaming length, eq.(2.26) for σ_{DM} , and eq.(2.22),

$$r_{lin} = \frac{l_{fs}}{\sqrt{3}} = \frac{269.5}{Z^{\frac{1}{3}}} \text{ kpc} = 125.1 \left(\frac{10}{Z} \right)^{\frac{1}{3}} \text{ kpc} \quad \text{and} \quad \frac{1}{\sigma_{DM}^2} = 2.34 \cdot 10^{13} Z^{\frac{2}{3}} \quad . \quad (2.27)$$

The velocity dispersion $\sigma_{DM} \sim 10^{-7}$ is very small since it does not take into account the self-gravity contrary to $\sigma_s \sim 10^{-5}$. σ_{DM} is just the primordial velocity dispersion redshifted today.

The linearized Boltzmann-Vlasov equation provides a single galaxy configuration with characteristic size $r_{lin} \sim l_{fs}$. This is an universal length and it corresponds to the largest galaxy sizes eq.(1.1), $r_{lin} \gtrsim r_0$. Non-linear effects would give a variety of galaxy configurations with smaller masses and sizes.

Inserting r_{lin} and $1/\sigma_{DM}^2$ eq.(2.27) and n_s eq.(2.10) in eq.(2.20) yields for the density profile and the surface density:

$$\rho_{lin}(r) = (5.826 \text{ Mev})^3 \frac{Z^{n_s/6}}{r} \int_0^\infty \gamma^{n_s/2-1} \log\left(\hat{c} Z^{\frac{1}{3}} \gamma\right) \sin\left(\gamma \frac{r}{r_{lin}}\right) T(\gamma) d\gamma, \quad (2.28)$$

$$\mu_{0D} = r_{lin} \rho_{lin}(0) = (5.826 \text{ Mev})^3 Z^{n_s/6} \int_0^\infty \gamma^{n_s/2} \log\left(\hat{c} Z^{\frac{1}{3}} \gamma\right) T(\gamma) d\gamma. \quad (2.29)$$

where $n_s/2 - 1 = -0.518$, $n_s/2 = 0.482$, $n_s/6 = 0.160$ and $\hat{c} = 43.6$.

We plot in fig. 2 $\rho_{lin}(r)/\rho_{lin}(0)$ vs. $x \equiv r/r_{lin}$ for Fermions (FD) and Bosons (BE) decoupling ultrarelativistically and for particles decoupling non-relativistically [Maxwell-Boltzmann statistics (MB)]. We choose for the plots the value $Z = 10$ both suggested by N -body simulations [24] and by the observations for μ_{0D} [eq.(1.2) and Table I]. Anyway, the Z dependence of $\rho_{lin}(r)/\rho_{lin}(0)$ vs. x is very mild, through $\log Z$, as shown by eqs.(2.28)-(2.29). The displayed profiles are clearly **cored**, as expected, since $T(k)$ decays for $k > k_{fs} \sim 1/r_{lin}$. Moreover, the profile eq.(2.28) is flat at $r = 0$ with a negative concavity at $r = 0$, namely $\rho'_{lin}(0) = 0$ and $\rho''_{lin}(0) < 0$. More explicitly,

$$\frac{\rho_{lin}(r)}{\rho_{lin}(0)} \underset{r \ll r_{lin}}{\approx} 1 - \frac{x^2}{6} \frac{\int_0^\infty \gamma^{2.482} \log\left(\hat{c} Z^{\frac{1}{3}} \gamma\right) T(\gamma) d\gamma}{\int_0^\infty \gamma^{0.482} \log\left(\hat{c} Z^{\frac{1}{3}} \gamma\right) T(\gamma) d\gamma} + \mathcal{O}(x^4) = 1 + \frac{r^2}{2} \frac{\rho''_{lin}(0)}{\rho_{lin}(0)} + \mathcal{O}(x^4).$$

We display in Table I the values of $r_{lin} \rho_{lin}(0)$ and $r_{lin}^2 \rho''_{lin}(0)/\rho_{lin}(0)$ for the three particle statistics. We find that $\rho_{lin}(0)_{BE} > \rho_{lin}(0)_{FD} > \rho_{lin}(0)_{MB}$. The more peaked density profile is the one for bosons and the more shallow is the non-relativistic one. The fermions profile being in-between the two other profiles.

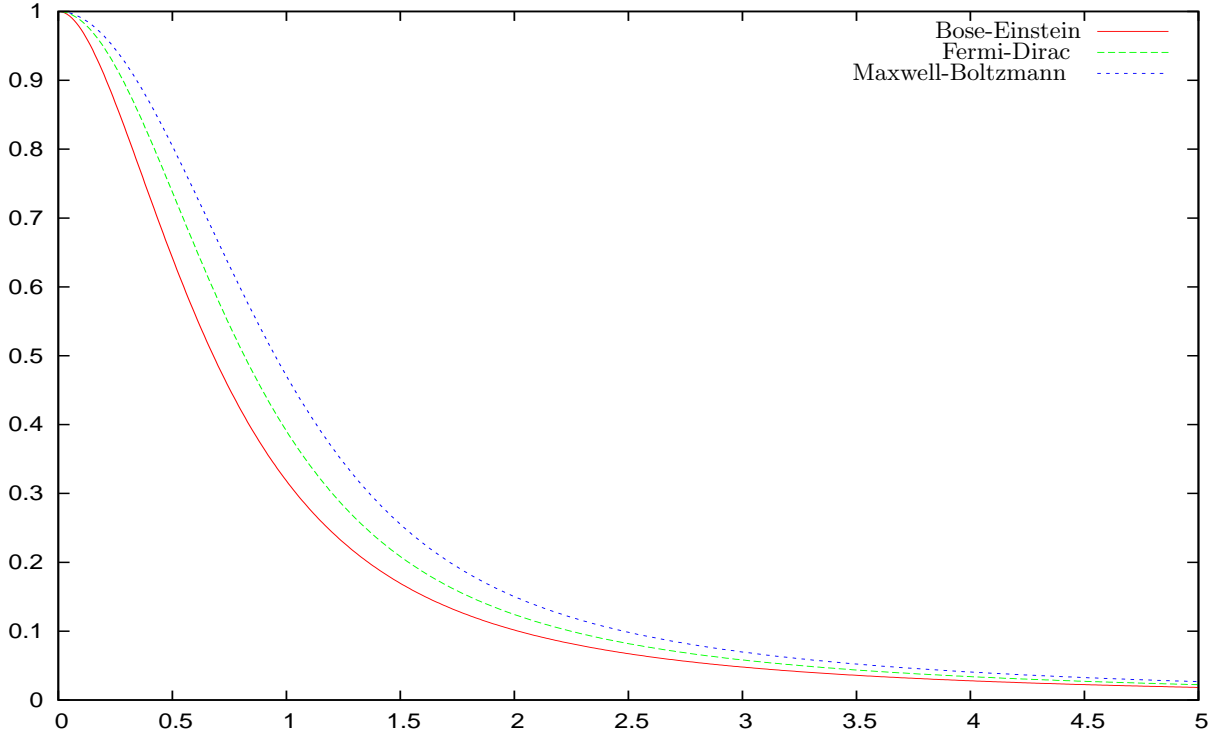


FIG. 2: The profiles $\rho_{lin}(r)/\rho_{lin}(0)$ vs. x , where $x \equiv r/r_{lin}$ for Fermions and Bosons decoupling ultrarelativistically and for particles decoupling non-relativistically (Maxwell-Boltzmann statistics). The bosons profile is the more peaked, the MB profile is the shallowest and the fermions profile is lying in-between.

The value of the surface density $r_0 \rho(0)$ should be **universal**. This is why we **identify** $r_{lin} \rho_{lin}(0)$ computed for a spherically symmetric solution of the linearized Boltzmann-Vlasov equation for self-gravitating DM with the

Particle Statistics	$\mu_{0D} = r_{lin} \rho_{lin}(0)$	$r_{lin}^2 \rho_{lin}''(0)/\rho_{lin}(0)$
Bose-Einstein	$(16.71 \text{ MeV})^3 (Z/10)^{0.16}$	-5.50
Fermi-Dirac	$(15.65 \text{ MeV})^3 (Z/10)^{0.16}$	-2.74
Maxwell-Boltzmann	$(14.73 \text{ MeV})^3 (Z/10)^{0.16}$	-1.83

TABLE I: Values obtained of the surface density $\mu_{0D} = r_{lin} \rho_{lin}(0)$ for Fermions and Bosons decoupling ultrarelativistically and for particles decoupling non-relativistically (Maxwell-Boltzmann statistics). [The exponent of Z originates in the primordial power $n_s/6 = 0.16$]. The comparison of these theoretical results for $\mu_{0D} = r_{lin} \rho_{lin}(0)$ with the observational value eq.(1.2) indicates that $Z \sim 10 - 100$ and therefore that the DM particle mass is in the keV range [see eq.(2.25)]. In any case, the agreement between the linear theory and the observations is **remarkable**.

observed value eq.(1.1). One representative solution is enough to obtain the value of the surface density but a more general treatment for non-spherically symmetrical solutions of the **non-linear** Boltzmann-Vlasov equation (and including also baryonic matter) would be necessary to prove the universality of $r_0 \rho(0)$.

We can estimate the mass of the galaxies obtained in the linear approximation from eqs.(2.27)-(2.28) as

$$M \sim r_{lin}^3 \rho_{lin}(0) = 1.8 \cdot 10^{14} M_\odot \left(\frac{10}{Z} \right)^{\frac{4-n_s}{6}}, \quad \frac{4-n_s}{6} \simeq 0.506.$$

We obtain mass values in the upper range of the observations, as expected.

Notice the scaling of the linear profile $\rho_{lin}(r)$ obtained here eq.(2.28) with the primordial spectral index n_s : $\rho_{lin}(r)$ decreases as $r^{-1-n_s/2} = r^{-1.482}$ for $r \gg r_{lin}$. The value of this exponent is in agreement with the universal empirical behaviour recently put forward in ref.[6] $r^{-1.6 \pm 0.4}$. For larger scales we would expect that the contribution from small k modes where nonlinear effects are dominant will give the customary r^{-3} tail.

We find that the surface density computed from the linearized Boltzmann-Vlasov equation reproduces very well the observed value of the energy scale eq.(1.2) depending on the particle statistics. Nonlinear effects should improve the theoretical values of the surface density $\mu_{0D} = r_{lin} \rho_{lin}(0)$ in Table I including the contributions from large scale modes. Notice from eq.(2.29) that the theoretical μ_{0D} has a **mild** dependence on Z , the only parameter here which is not known with precision.

Anyhow, the agreement between the linear theory and the observations is already **remarkable**. The comparison of our theoretical values for μ_{0D} displayed in Table I and the observational value eq.(1.2) indicates that $Z \sim 10 - 100$. This implies that the DM particle mass is in the keV range eq.(2.23) and (2.25).

Notice that r_{lin} in eq.(2.27) decreases with Z as $Z^{-\frac{1}{3}}$, while $\rho_{lin}(0)$ in eq.(2.29) grows with Z as $Z^{(n_s+2)/6} \ln Z = Z^{0.493} \ln Z$. Cusp profiles would thus correspond to the unphysical limit $Z \rightarrow \infty$. The DM particle mass m grows as $Z^{\frac{1}{4}}$ according to eq.(2.23) (or as $Z^{\frac{1}{3}}$ for DM particles decoupling non-relativistically [15]). Cusped profiles are thus associated to heavy DM particles with a huge mass m well above the physical keV scale. For example, wimps at $m = 100 \text{ GeV}$ and $T_d = 5 \text{ GeV}$ [27] would require $Z \sim 10^{24}$ [15]. Such heavy DM candidates are in conflict with the observed surface density eq.(1.2), since eq.(1.2) indicates that $Z \sim 10 - 100$ and therefore that the DM particle mass is in the keV range [see eq.(2.25)]. Therefore, our present results for $\mu_{0D} = r_{lin} \rho_{lin}(0)$ provide further evidence for the mass scale of the DM particle being in the keV scale.

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